Workshop on Mathematical Challenges for Sustainability

White paper on energy systems analysis and economics

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The emerging vision of energy systems introduces a wide range of problems in the control and valuation of energy and energy assets and the design of public policies fostering cleaner technologies and more reliable production and transmission. As we move toward greater dependence on renewable energies such as wind and solar, combined with dynamic pricing of electricity and markets for carbon offsets, we encounter fundamental mathematical and algorithmic challenges to solve the stochastic optimization problems that arise in these settings. Major classes of problems include

- Energy policy – How should we design market mechanisms to control green house gases? How do we design investment tax credits to encourage market adoption and innovation?
- Strategic planning - How do we invest R&D resources in different energy technologies? How do we plan investments in infrastructure such as ethanol, wind, solar, "clean coal" and geothermal? Do we install 220V charging stations, or wait for 480V technologies to evolve?
- Energy and finance – Both long-term and short-term energy investments incur significant risk, which can be mitigated using modern hedging strategies. How do we design these instruments, and what are the implications in terms of managing risk?
- Energy systems analysis – We need to design and control delivery systems, which means understanding how to optimize the networks that generate, transmit, move and store energy.

A major theme that cuts across these topics involves making decisions in the presence of different forms of uncertainty. We need to understand the nature of uncertainty, and how to make decisions in the presence of different types of uncertainty (coarse grained, fine grained, quantifiable and speculative). We need advances in the tools for solving stochastic optimization problems, and we need frameworks for modeling more complex forms of risk.

The research spans fundamental theoretical research in stochastic analysis to gain an understanding of the properties of these problems, research into new algorithmic strategies to solve these problems, and research to take advantage of the power of high performance computing, including massively parallel architectures and cloud computing, which introduce new opportunities and challenges for stochastic optimization.

There are numerous examples of these research challenges. These cut across the themes described above, but below is a partial list organized roughly between the general areas of policy and economics, and energy systems analysis:

Energy policy and economics

- How much should we rely on market mechanisms to control Green House Gas (GHG) emissions, and how should we design these markets to fully capture the environment benefits while minimizing the social costs and maintaining a competitive edge? Realistic models need to be studied carefully in order to understand the real impacts of untested policies, and the possible economic consequences of "leakage." Only the solution of rather sophisticated stochastic optimization problems (bringing to bear the economics of real options and engineering constraints) will help us understand when it is worth abandoning a project or switching production technologies.
- Environmental economics has experienced a quantum leap in growth over the last twenty years, and the need for large scale optimization problems incorporating stochastic factors is now recognized as a sine-qua-non condition for the transition from the analysis of illustrative toy models to realistic case studies which could be used for policy making purposes.
- While competitive equilibriums and cooperative games are relatively well understood in deterministic settings, more realistic models where demand and costs of production appear as random factors are needed to understand the workings of deregulated energy markets, and the strategic behavior of non-cooperative agents competing for government subsidies and market shares.
- R&D portfolio optimization – What technologies should DOE invest in to meet goals for renewables within specific time frames, given the tremendous uncertainty about breakthroughs in materials science, technology and manufacturing?
- Investments in different technologies such as nuclear, natural gas, wind and solar require models that capture the fine-grained variability in wind and solar; hourly, daily and seasonal pricing patterns; and weekly and seasonal rainfall patterns. We need to capture both the variability and the uncertainty in these dynamic prices, producing stochastic optimization problems with complex dynamics.
- Utilities need to plan investments in the grid, generation and storage. This cannot be done without a deep understanding of the economics of energy. These capital-intensive operations are not possible without the involvement of financial institutions, and the understanding of hedging strategies. Goldman Sachs and Morgan Stanley have been major players in this arena where they became the masters of a new breed of institutions in dealing simultaneously with financial and financial energy assets. But their business model has been emulated on a large scale, not without spectacular failures and losses.
- Most of the financial institutions (major banks as well as hedge funds) participating in energy markets now understand that when they purchase (or at least lease) physical assets

such as power plants, storage facilities, pipelines, tankers, they need to hedge the associated risks with financial instruments, and conversely, hedge financial positions in the energy markets by purchasing or leasing physical assets. In fact the duality between purely financial and purely physical assets is now well understood and taken full advantage of by financial institutions and energy companies alike. Because of the sheer magnitude of their economic and environmental impacts, the intellectual challenges raised by the management of these mixed portfolios and the risks they carry are some of the most exciting recent developments in financial engineering.

Energy systems analysis

- Utilities use large-scale integer programming models to capture the ability of different types of energy generators (nuclear, coal, natural gas, hydro, etc) to respond to the variability of demand, wind and solar. Current algorithms can only handle deterministic models, and even these require hundreds of hours on a modern computer to capture an entire year, which is needed to understand the behavior over different seasons.
- Buildings can combine energy from the grid, solar panels, wind turbines, storage devices and local diesel generators to meet their needs. Building managers need to minimize cost by controlling these resources efficiently, while also minimizing the risk of being exposed to high prices or using the diesel generator in a way that violates environmental regulations.
- Utilities would like to use the batteries in electric vehicles as a source of backup power during peak periods such as hot afternoons. This requires first knowing how to optimally charge and discharge batteries in a way that adapts to the specific behavioral patterns of drivers.
- Companies want to invest in energy technologies which reduce their carbon footprint, but the value of these technologies depend on the price of carbon, which may be highly volatile in the presence of political risk, carbon pricing and trading strategies, and moral hazard. We need robust models to understand the sources of carbon price volatility, and methods for planning investments which minimize all the risks involved.
- Based on the principle of the Clean Development Mechanism (CDM) and the Joint Initiative (JI) companies are still selling carbon offsets by maintaining forests that would otherwise be converted to agriculture. But this requires a rigorous verification process (e.g. measuring the actual amount of carbon being absorbed) and in order to preserve the integrity of the process, we need to develop efficient sampling methodologies to verify the status of these resources.

This partial list of problems span real-time control, tactical and strategic planning, where we are also interested in valuing assets and understanding the risks of different strategies. All of these problems produce complex stochastic optimization problems that cannot be solved analytically, and for which robust algorithms with provable guarantees do not exist.

Most of the problems can be formulated as a stochastic optimization problem with the general form

$$\max_{\pi \in \Pi} E\left\{ \sum_{t=0}^{\infty} \gamma^t C(S_t, X^\pi(S_t)) \right\} \tag{1}$$

where S_t is the information available at time *t*, and $X^\pi(S_t)$ is a policy that returns a decision (or control) x_t. Depending on the characteristics of the problem, we might use a lookahead policy, a parametric or nonparametric policy, or a policy based on a value function approximation using either a parametric or nonparametric policy. In principle, all three strategies can be used to produce a near-optimal policy, but practical algorithms with provable guarantees in finite time do not exist for all but the simplest problems. Lookahead policies which capture uncertainty can be extremely slow; general purpose policy function approximation methods for higher dimensional problems do not exist; and algorithms based on value function approximations remain an attractive but frustrating strategy where robust algorithms exist only for special cases.

The situation gets even worse. The objective function in (1) assumes linear, additive rewards. This does not apply to all problems. For example, a building manager has to make daily decisions about how to meet its energy needs while minimizing the risk of exceeding monthly limits on the use of external diesel generators, and energy investment decisions have to meet specific targets for renewables within a time frame.

An additional complication arises when the expectation in (1) is poorly defined (how do we capture the likelihood of a breakthrough in solar panels) or undefined, which is what happens in the presence of heavy-tailed price processes which exhibit infinite variance. We need new paradigms for stochastic optimization for solving these emerging classes of problems.

A major algorithmic strategy used in stochastic optimization is the use of functional approximations. These arise in pure and hybrid policy search algorithms, and are especially prominent in approximate dynamic programming when using value function approximations. The fields of statistics and machine learning are replete with algorithms, but stochastic search imposes special demands. For example, since we are optimizing, we are not interested in replicating the entire function, which means we are willing to live with simpler, lower dimensional representations of regions of the function which are less interesting, but we would like to evolve to higher dimensional representations in regions of high interest, which may be poorly behaved.

A related challenge arises when trying to approximate medium to high dimensional problems. Classical approximation methods struggle with problems with as few as five dimensions. Separable approximations scale to very high-dimensional problems, but at the expense of errors introduced by using a separable approximation. Most high-dimensional problems exhibit lower order interactions, but this introduces the challenge of identifying these interactions.

These challenges represent practical barriers to the design of robust, effective algorithms. These algorithms are needed by policy makers and planners to design robust systems, and to understand the economics of how these systems will work in practice. Utilities need these algorithms to operate their networks efficiently and reliably. Manufacturers need these algorithms to determine when batteries in electric vehicles should be charged. Grid operators need to know how to price electricity to balance demand across the network. And investors need to understand the risk of alternative energy supplies.